Indian Statistical Institute
Semestral Examination
Topology I - MMath I

Max Marks: 60 Time: 180 minutes.

Answer all questions. You may use Theorems stated/proved in the class after correctly stating them. You may use results not discussed in the class only after proving them.

(1) (a) When do you say a space is regular? Give an example of a Hausdorff space that is not regular.
(b) Is $[0,1]^{\mathbb{N}}$ compact in the uniform metric topology? In the box topology?
(c) Let X be an infinite countable set. Show that there does not exist a topology τ on X that is both connected and normal.
(d) Is $\mathbb{R}$ with the lower limit topology second countable? [5+5+5+5]

(2) (a) Let X be a metric space with metric d. Show that $d : X \times X \longrightarrow \mathbb{R}$ is continuous. Let X' denote a space having the same underlying set as X. If $d : X' \times X' \longrightarrow \mathbb{R}$ is continuous, then show that the topology of X' is finer than the topology of X.
(b) Let X be metrizable. Show that the following are equivalent.
(i) X is bounded under every metric that gives the topology of X.
(ii) Every continuous function $f : X \longrightarrow \mathbb{R}$ is bounded.
(iii) X is limit point compact. [10+10]

(3) (a) Define the notion of homotopy of maps. Show that being homotopic is an equivalence relation on the set of maps from X to Y.
(b) Show that every map $f : S^1 \longrightarrow S^1$ is homotopic to a map g such that $g(1) = 1$.
(c) Suppose $f : S^n \longrightarrow S^n$ is a map such that $f(x) \neq x$ for all $x \in S^n$. Show that $f \sim a$ where $a : S^n \longrightarrow S^n$ is the map $a(x) = -x$. [6+7+7]